CHAPTER 24 GAUSS'S LAW

ActivPhysics can help with these problems: Activities 11.4-11.6

Section 24-1: Electric Field Lines

Problem

1. What is the net charge shown in Fig. 24-39? The magnitude of the middle charge is 3 mC.

Solution

The number of lines of force emanating from (or terminating on) the positive (or negative) charges is the same (14 in Fig. 24-39), so the middle charge is -3 mC and the outer ones are +3 mC. The net charge shown is therefore 3 + 3 - 3 = 3 mC. This is reflected by the fact that 14 lines emerge from the boundary of the figure.



FIGURE 24-39 Problem 1 Solution.

Problem

2. A charge +2q and a charge -q are near each other. Sketch some field lines for this charge distribution, using the convention of eight lines for a charge of magnitude q.

Solution

The sketch is similar to Fig. 24-4(f) with twice the number of lines of force.

Problem

3. Two charges +q and a charge -q are at the vertices of an equilateral triangle. Sketch some field lines for this charge distribution.

Solution

(The sketch shown follows the text's convention of eight lines of force per charge magnitude q.)



Problem 3 Solution.

Problem

4. The net charge shown in Fig. 24-40 is +Q. Identify each of the charges A, B, C shown.



FIGURE 24-40 Problem 4.

Solution

From the direction of the lines of force (away from positive and toward negative charge) one sees that *A* and *C* are positive and *B* is a negative charge. Eight lines of force terminate on *B*, eight originate on *C*, but only four originate on *A*, so the magnitudes of *B* and *C* are equal, while the magnitude of *A* is half that value. Thus, $Q_C = -Q_B = 2Q_A$. The total charge is $Q = Q_A + Q_B + Q_C = Q_A$, so $Q_C = 2Q = -Q_B$.

Section 24-2: Electric Flux

Problem

5. A flat surface with area 2.0 m² is in a uniform electric field of 850 N/C. What is the electric flux through the surface when it is (a) at right angles to the field, (b) at 45° to the field, and (c) parallel to the field?

Solution

(a) When the surface is perpendicular to the field, its normal is either parallel or anti-parallel to **E**. Then Equation 24-1 gives $\Phi = \mathbf{E} \cdot \mathbf{A} = EA \cos(0^{\circ} \text{ or } 180^{\circ}) = \pm(850 \text{ N/C})(2 \text{ m}^2) = \pm1.70 \text{ kN} \cdot \text{m}^2/\text{C}$. (b) $\Phi = \mathbf{E} \cdot \mathbf{A} = EA \cos(45^{\circ} \text{ or } 135^{\circ}) = \pm(1.70 \text{ kN} \cdot \text{m}^2/\text{C})(0.866) = \pm1.20 \text{ kN} \cdot \text{m}^2/\text{C}$. (c) $\Phi = EA \cos 90^{\circ} = 0$.

Problem

6. What is the electric field strength in a region where the flux through 1.0 cm \times 1.0 cm flat surface is 65 N \cdot m²/C, if the field is uniform and the surface is at right angles to the field?

Solution

The magnitude of the flux through a flat surface perpendicular to a uniform field is $|\Phi| = EA$ (see solution to part (a) of the previous problem). Thus $E = |\Phi| = A = (65 \text{ N} \cdot \text{m}^2/\text{C} = (10^{-4} \text{ m}^2) = 650 \text{ kN/C}.$

Problem

7. A flat surface with area 0.14 m² lies in the *x*-*y* plane, in a uniform electric field given by $\mathbf{E} = 5.1\hat{\mathbf{i}} + 2.1\hat{\mathbf{j}} + 3.5\hat{\mathbf{k}}$ kN/C. Find the flux through this surface.

Solution

The surface can be represented by a vector area $\mathbf{A} = (0.14 \text{ m}^2)(\pm \hat{\mathbf{k}})$. (Since the surface is open, we have a choice of normal to the *x*-*y* plane.) Then

 $\Phi = \mathbf{E} \cdot \mathbf{A} = \pm \mathbf{E} \cdot \hat{\mathbf{k}} \times (0.14 \text{ m}^2) = \pm E_z (0.14 \text{ m}^2) = \pm (3.5 \text{ kN/C})(0.14 \text{ m}^2) = \pm 490 \text{ N} \cdot \text{m}^2/\text{C}.$ (Only the *z* component of the field contributes to the flux through the *x*-*y* plane.)

Problem

8. The electric field on the surface of a 10-cm-diameter sphere is perpendicular to the sphere and has magnitude 47 kN/C. What is the electric flux through the sphere?

Solution

For a sphere, with **E** parallel or anti-parallel to $\hat{\mathbf{n}}$, Equation 24-2 gives $\Phi = \pm 4\mathbf{p}r^2E = \pm 4\mathbf{p}(0.1 \text{ m/2})^2(47 \text{ kN/C}) = \pm 1.48 \text{ kN} \cdot \text{m}^2/\text{C}.$

Problem

9. What is the flux through the hemispherical open surface of radius *R* shown in Fig. 24-41? The uniform field has magnitude *E*. *Hint*: Don't do a messy integral! Imagine closing the surface with a flat, circular piece across the open end. What would be the flux through the entire closed surface? And what's the flux through the flat end? So what's the answer?

Solution

All of the lines of force going through the hemisphere also go through an equitorial disk covering its edge in Fig. 24-41. Therefore, the flux through the disk (normal in the direction of **E**) equals the flux through the hemisphere. Since **E** is uniform, the flux through the disk is just pR^2E . (Note: Gauss's Law gives the same result, since the flux through the closed surface, consisting of the hemisphere plus the disk, is zero. See Section 24-3.)



FIGURE 24-41 Problem 9 Solution.

Problem

10. The electric field shown in Fig. 24-42 is given by $\mathbf{E} = E_0(y=a)\hat{\mathbf{k}}$, where E_0 and *a* are constants. Find the flux through the square of side *a* shown.

Solution

Use area elements described in the solution to Problem 65 and Equation 24-2: $\Phi = \mathbf{Z} \cdot d\mathbf{A} = \mathbf{Z} (E_0 y = a) \hat{\mathbf{k}} \cdot (\pm a dy \hat{\mathbf{k}}) = \pm E_0 \mathbf{Z} y dy = \pm \frac{1}{2} E_0 a^2$.



FIGURE 24-42 Problems 10 and 65 Solution.

Section 24-3: Gauss's Law

Problem

11. What is the electric flux through each closed surface shown in Fig. 24-43?



FIGURE 24-43 Problem 11.

Solution

From Gauss's law, $\Phi = q_{\text{enclosed}} = e_0$. For the surfaces shown, this is (a) $(q - 2q) = e_0 = -q = e_0$, (b) $-2q = e_0$, (c) and (d) 0.

Problem

12. A 6.8-mC charge and a -4.7 mC charge are inside an uncharged sphere. What is the electric flux through the sphere?

Solution

Since the sphere encloses both charges (and none other) the flux through it is $q_{\text{enclosed}} = \mathbf{e}_0 = (6.8 - 4.7)\mathbf{m}C = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}) = 237 \text{ kN} \cdot \text{m}^2/\text{C}.$

Problem

13. A 2.6-**m**C charge is at the center of a cube 7.5 cm on each side. What is the electric flux through one face of the cube? *Hint:* Think about symmetry, and don't do an integral.

Solution

The symmetry of the situation guarantees that the flux through one face is $\frac{1}{6}$ the flux through the whole cubical surface, so

$$\Phi_{\text{face}} = \frac{1}{6} \sum \mathbf{E} \cdot d\mathbf{A} = q_{\text{enclosed}} = 6\mathbf{e}_0 = (2.6 \text{ mC}) = 6(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 49.0 \text{ kN} \cdot \text{m}^2/\text{C}.$$

Problem

14. If the charge in the preceding problem is still inside the cube but not at the center, (a) what is the flux through the *entire* cube? (b) Could you still calculate the flux through one face without doing an integral?

Solution

(a) The flux through the closed cubical surface is the same as before, namely $6 \times 49.0 = 294 \text{ kN} \cdot \text{m}^2/\text{C}$, but (b) the flux through any particular face depends on the location of the charge, if it's not at the center.

Problem

15. A dipole consists of two charges ±6.1 mC located 1.2 cm apart. What is the electric flux through each surface shown in Fig. 24-44?



FIGURE 24-44 Problem 15.

Solution

It follows from Gauss's law that (a) $\Phi_a = +q = e_0 = (6.1 \text{ mC}) = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 689 \text{ kN} \cdot \text{m}^2/\text{C}$, (b) $\Phi_b = -\Phi_a$, and (c) $\Phi_c = 0$.

Problem

16. The electric field in a certain region is given by $\mathbf{E} = 40x\mathbf{\hat{i}} \text{ N/C}$, with *x* in meters. What is the volume charge density in the region? *Hint*: Apply Gauss's law to a cube 1 meter on a side.

Solution

It is most convenient to orient a closed cubical gaussian surface (of side length ℓ) with faces parallel to the coordinate planes. Since $\mathbf{E} = E_x \hat{\mathbf{i}}$, there is non-zero flux only through the two faces at x_1 and x_2 , whose outward unit normals are $-\hat{\mathbf{i}}$ and $+\hat{\mathbf{i}}$, respectively. Gauss's law applied to the cube gives $\sum_{\text{curve}} \mathbf{E} \cdot d\mathbf{A} = (40 \text{ N/m} \cdot \text{C})(x_2 - x_1)\ell^2 = q_{\text{enclosed}} = \boldsymbol{e}_0$. Since $x_2 - x_1 = \ell$, the charge density, which is uniform, is $\mathbf{r} = q_{\text{enclosed}} = \ell^3 = 40\boldsymbol{e}_0 \text{ N/m} \cdot \text{C} = (40 \text{ N/m} \cdot \text{C}) \times (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 3.54 \times 10^{-10} \text{ C/m}^3$.

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Problem 16 Solution.

Section 24-4: Using Gauss's Law

Problem

17. The electric field at the surface of a uniformly charged sphere of radius 5.0 cm is 90 kN/C. What would be the field strength 10 cm from the surface?

Solution

The electric field due to a uniformly charged sphere is like the field of a point charge for points outside the sphere, i.e., $E(r) \gg 1 = r^2$ for $r \ge R$. Thus, at 10 cm from the surface, r = 15 cm and $E(15 \text{ cm}) = (5 = 15)^2 E(5 \text{ cm}) = (90 \text{ kN/C}) = 10 \text{ kN/C}$.

Problem

18. A solid sphere 25 cm in radius carries 14 **m**C, distributed uniformly throughout its volume. Find the electric field strength (a) 15 cm, (b) 25 cm, and (c) 50 cm from the sphere's center.

Solution

Example 24-1 shows that (a) at 15 cm = r < R = 25 cm, $E = kQr = R^3 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(14 \text{ mC})(15 \text{ cm}) = (25 \text{ cm})^3 = 1.21 \text{ MN/C}$, (b) at r = R, $E = kQ = R^2 = (\frac{5}{3})(1.21 \text{ MN/C}) = 2.02 \text{ MN/C}$, and (c) at r = 2R > R, $E = kQ = (2R)^2 = (\frac{1}{4})(2.02 \text{ MN/C}) = 504 \text{ kN/C}$.

Problem

19. A crude model for the hydrogen atom treats it as a point charge +e (the proton) surrounded by a uniform cloud of negative charge with total charge -e and radius 0.0529 nm. What would be the electric field strength inside such an atom, halfway from the proton to the edge of the charge cloud?

Solution

At half the radius of the electron cloud, the field strength due to the cloud (a uniformly charged spherical volume) is given in Example 24-1: $E_e = k(-e)(\frac{1}{2}R)=R^3 = -ke=2R^2$. The field strength due to the proton (a point charge) at the same distance is $E_p = ke=(\frac{1}{2}R)^2 = 4ke=R^2$. (The electron's field is radially inward, negative, and the proton's is radially outward, positive.) The total field strength is

$$E = E_e + E_p = -ke = 2R^2 + 4ke = R^2 = 7ke = 2R^2 = 7(1.44 \times 10^{-9} \text{ N} \cdot \text{m}^2/\text{C}) = 2(5.29 \times 10^{-11} \text{ m})^2 = 1.80 \times 10^{12} \text{ N/C}.$$

Problem

20. Positive charge is spread uniformly over the surface of a spherical balloon 70 cm in radius, resulting in an electric field of 26 kN/C at the balloon's surface. Find the field strength (a) 50 cm from the balloon's center and (b) 190 cm from the center. (c) What is the net charge on the balloon?

Solution

(a) Inside a uniformly charged spherical shell, the electric field is zero (see Example 24-2). (b) Outside, the field is like that of a point charge, with total charge at the center, so $E(190 \text{ cm}) = E(70 \text{ cm})(70=190)^2 = (0.136)(26 \text{ kN/C}) = 3.53 \text{ kN/C}$. (c) Using the given field strength at the surface, we find a net charge $Q = ER^2 = k = (26 \text{ kN/C})(0.7 \text{ m})^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) = 1.42 \text{ mC}$.

Problem

21. A 10-nC point charge is located at the center of a thin spherical shell of radius 8.0 cm carrying -20 nC distributed uniformly over its surface. What are the magnitude and direction of the electric field (a) 2.0 cm, (b) 6.0 cm, and (c) 15 cm from the point charge?

Solution

The total electric field, the superposition of the fields due to the point charge and the spherical shell, is spherically symmetric about the center. Inside the shell (r < R = 8 cm), its field is zero, so the total field is just due to the 10 mC point charge. Outside (r > R), the shell's field is like that of a point charge of -20 mC at the same central location as the 10 mC charge. (This situation is described in Example 24-3.) (a) and (b) For r = 2 cm or 6 cm < R, $E = kq_{pt} = r^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10 \text{ nC}) = (2 \text{ cm or } 6 \text{ cm})^2 = 225 \text{ kN/C or } 25.0 \text{ kN/C}$, respectively, directed radially outward. (c) For r = 15 cm > R, $E = k(q_{pt} + q_{shell}) = r^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10 \text{ nC} - 20 \text{ nC}) = (-4.00 \text{ kN/C})$, directed radially inward.

Problem

22. A solid sphere 2.0 cm in radius carries a uniform volume charge density. The electric field 1.0 cm from the sphere's center has magnitude 39 kN/C. (a) At what other distance does the field have this magnitude? (b) What is the net charge on the sphere?

Solution

(a) Referring to Example 24-1, we see that at $r = \frac{1}{2}R$, $E = kQ(\frac{1}{2}R)=R^3 = kQ=2R^2$. This is also the field strength outside the sphere at a distance $r = \sqrt{2}R = \sqrt{2}(2 \text{ cm}) = 2.83 \text{ cm}$. (b) Using the given field strength at $r = \frac{1}{2}R$, we find $Q = 2R^2 E = k = 2(2 \text{ cm})^2 (39 \text{ kN/C}) = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) = 3.47 \text{ nC}$.

Problem

23. A point charge -2Q is at the center of a spherical shell of radius *R* carrying charge *Q* spread uniformly over its surface. What is the electric field at (a) $r = \frac{1}{2}R$ and (b) r = 2R? (c) How would your answers change if the charge on the shell were doubled?

Solution

The situation is like that in Problem 21. (a) At $r = \frac{1}{2}R < R$ (inside shell), $E = E_{pt} + E_{shell} = k(-2Q) = (\frac{1}{2}R)^2 + 0 = -8 kQ = R^2$ (the minus sign means the direction is radially inward). (b) At r = 2R > R (outside shell), $E = E_{pt} + E_{shell} = k(-2Q + Q) = (2R)^2 = -kQ = 4R^2$ (also radially inward). (c) If $Q_{shell} = 2Q$, the field inside would be unchanged, but the field outside would be zero (since $q_{shell} + q_{pt} = 2Q - 2Q = 0$).

Problem

24. A spherical shell of radius 15 cm carries 4.8 mC, distributed uniformly over its surface. At the center of the shell is a point charge. (a) If the electric field at the surface of the sphere is 750 kN/C and points outward, what is the charge of the point charge? (b) What is the field just inside the shell?

Solution

(a) As in the previous solution, the field strength at the surface of the shell (r = R) is $E_{pt} + E_{shell} = k(q_{pt} + q_{shell}) = R^2$, hence $q_{pt} = [(15 \text{ cm})^2 (750 \text{ kN/C}) = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)] - 4.8 \text{ mC} = -2.93 \text{ mC}$. (b) Just inside the shell, at r = 15 cm - d(where $d \ge 15 \text{ cm}$), the field is due to the point charge only: $E = k(-2.93 \text{ mC}) = (15 \text{ cm} + d)^2 \frac{14}{2} - 1.17 \text{ MN/C}$, directed radially inward.

Problem

25. A spherical shell 30 cm in diameter carries a total charge 85 mC distributed uniformly over its surface. A 1.0-mC point charge is located at the center of the shell. What is the electric field strength (a) 5.0 cm from the center and (b) 45 cm from the center? (c) How would your answers change if the charge on the shell were doubled?

Solution

(a) The field due to the shell is zero inside, so at r = 5 cm, the field is due to the point charge only. Thus, $\mathbf{E} = kq\hat{\mathbf{r}}=r^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1 \text{ mC})\hat{\mathbf{r}}=(0.05 \text{ m})^2 = (3.60 \times 10^6 \text{ N/C})\hat{\mathbf{r}}$. (b) Outside the shell, its field is like that of a point charge, so at r = 45 cm, $\mathbf{E} = k(q + Q)\hat{\mathbf{r}}=r^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(86 \text{ mC})\hat{\mathbf{r}}=(0.45 \text{ m})^2 = (3.82 \times 10^6 \text{ N/C})\hat{\mathbf{r}}$. (c) If the charge on the shell were doubled, the field inside would be unaffected, while the field outside would approximately double, $E = k(1.0 \text{ mC} + 2 \times 85 \text{ mC})=(45 \text{ cm})^2 = 7.60 \text{ MN/C}$.

Problem

26. The thick, spherical shell of inner radius *a* and outer radius *b* shown in Fig. 24-45 carries a uniform volume charge density *r*. Find an expression for the electric field strength in the region a < r < b, and show that your result is consistent with Equation 24-7 when a = 0.

Solution

Use the result of Gauss's law applied to a spherically symmetric distribution, $E = q_{\text{enclosed}} = 4pe_0r^2$. For a < r < b in a spherical shell with charge density \mathbf{r} , $q_{\text{enclosed}} = \frac{4}{3}\mathbf{p}(r^3 - a^3)\mathbf{r}$, so $E = \mathbf{r}(r^3 - a^3) = 3\mathbf{e}_0r^2 = (\mathbf{r}=3\mathbf{e}_0)(r - a^3 = r^2)$. If $a \to 0$, Equation 24-7 for a uniformly charged spherical volume is recovered.



FIGURE 24-45 Problem 26 Solution.

Problem

27. How should the charge density within a solid sphere vary with distance from the center in order that the magnitude of the electric field in the sphere be constant?

Solution

Assume that r is spherically symmetric, and divide the volume into thin shells with $dV = 4pr^2 dr$. From Gauss's law and Equation 24-5,

$$E = \frac{1}{4pe_0r^2} \sum_{r} dV = \frac{1}{4pe_0r^2} \sum_{r} r(r') 4pr'^2 dr' = \frac{1}{e_0r^2} \sum_{r} rr'^2 dr'.$$

It can be seen that if $r(r') \gg 1 = r'$ then *E* is constant, but we can obtain the same result mathematically, by differentiation. If *E* is constant, dE = dr = 0. This implies

$$0 = \frac{d}{dr} \prod_{r} \frac{1}{r^2} \left[\sum_{r'} r'^2 dr' \right] = \frac{1}{r^2} \frac{d}{dr} \left[\sum_{r'} r'^2 dr' \right] + \left[\sum_{r'} r'^2 dr' \right] + \left[\sum_{r'} r'^2 dr' \right] = \frac{1}{r^2} r'(r)r^2 - \frac{2}{r^3} \sum_{r'} r'^2 dr',$$
$$r(r) = \frac{2}{r^3} \sum_{r'} r'(r')r'^2 dr'.$$

Since $r^{-2} \mathbf{Z} \mathbf{r}(r')r'^2 dr' = \mathbf{e}_0 E$ is a constant, by hypothesis, $\mathbf{r}(r) = 2\mathbf{e}_0 E = r \cdot \mathbf{r}$, as suspected. (Look up how to take the derivative of an integral in any calculus textbook.) Note that constant magnitude does not imply constant direction; $\mathbf{E} = E\hat{\mathbf{r}}$ is spherically symmetric, not uniform.

Problem

or

28. A long, thin wire carrying 5.6 nC/m runs down the center of a long, thin-walled, hollow pipe with radius 1.0 cm carrying -4.2 nC/m spread uniformly over its surface. Find the electric field (a) 0.50 cm from the wire and (b) 1.5 cm from the wire.

Solution

Assume that the electric field is approximately that from an infinitely long, cylindrically symmetric charge distribution, $E = \mathbf{1}_{enclosed} = 2\mathbf{p}\mathbf{e}_0 r$, where $\mathbf{1}_{enclosed}$ is the change inside a unit length of cylindrical Gaussian surface of radius *r* about the symmetry axis. (a) For r = 0.5 cm, between the wire and the pipe, $\mathbf{1}_{enclosed} = \mathbf{1}_{wire}$, and E = 2k(5.6 nC/m) = (0.5 cm) = 20.2 kN/C (positive, radially away from the axis of symmetry, i.e., the wire). (b) For r = 1.5 cm, $\mathbf{1}_{enclosed} = \mathbf{1}_{wire} + \mathbf{1}_{pipe}$, and E = 2k(5.6 - 4.2)(nC/m) = (1.5 cm) = 1.68 kN/C (in the same direction as the field in part (a)).

Problem

29. A long solid rod 4.5 cm in radius carries a uniform volume charge density. If the electric field strength at the surface of the rod (not near either end) is 16 kN/C, what is the volume charge density?

Solution

If the rod is long enough to approximate its field using line symmetry, we can equate the flux through a length ℓ of its surface (Equation 24-8) to the charge enclosed. The latter is the charge density (a constant) times the volume of a length ℓ of rod. Thus, $2\mathbf{p}R\ell E = q_{\text{enclosed}} = \mathbf{r}\mathbf{p}R^2\ell = \mathbf{e}_0$, or $\mathbf{r} = 2\mathbf{e}_0 E = R = 2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(16 \text{ kN/C}) = 6.29 \text{ mC/m}^3$. (This is the magnitude of \mathbf{r} , since the direction of the field at the surface, radially inward or outward, was not specified.)

Problem

30. The electric field strength outside a charge distribution and 18 cm from its center has magnitude 55 kN/C. At 23 cm

the field strength is 43 kN/C. Does the distribution have spherical or line symmetry?

Solution

For spherical symmetry, $E \gg r^{-2}$, and for line symmetry, $E \gg r^{-1}$. From the data supplied, $E \gg r^{-n}$ implies $(55=43) = (18=23)^{-n}$, or $n = \ln(55=43) = \ln(23=18) = 1.00$, in agreement with line symmetry.

Problem

31. An infinitely long rod of radius *R* carries a uniform volume charge density *r*. Show that the electric field strengths outside and inside the rod are given, respectively, by $E = rR^2 = 2e_0r$ and $E = rr = 2e_0$, where *r* is the distance from the rod axis.

Solution

The charge distribution has line symmetry (as in Problem 29) so the flux through a coaxial cylindrical surface of radius r (Equation 24-8) equals $q_{\text{enclosed}} = \mathbf{e}_0$, from Gauss's law. For r > R (outside the rod), $q_{\text{enclosed}} = \mathbf{rp}R^2\ell$, hence $E_{\text{out}} = \mathbf{rp}R^2\ell=2\mathbf{p}r\ell\mathbf{e}_0 = \mathbf{r}R^2=2\mathbf{e}_0r$. For r < R (inside the rod), $q_{\text{enclosed}} = \mathbf{rp}r^2\ell$, hence $E_{\text{in}} = \mathbf{rp}r^2\ell=2\mathbf{p}r\ell\mathbf{e}_0 = \mathbf{r}r=2\mathbf{e}_0$. (The field direction is radially away from the symmetry axis if $\mathbf{r} > 0$, and radially inward if $\mathbf{r} < 0$.)

Problem

32. Repeat Problem 26, assuming that Fig. 24-45 represents the cross section of a long, thick-walled pipe. Now the case a = 0 should be consistent with the result of Problem 31 for the interior of the rod.

Solution

Suppose that the pipe is long enough that the line symmetric result in Equation 24-8 can be used in Gauss's law. Then $E = q_{\text{enclosed}} = 2\mathbf{p}\mathbf{e}_0 r\ell$. For a < r < b, $q_{\text{enclosed}} = \mathbf{r}V = \mathbf{r}\mathbf{p}(r^2 - a^2)\ell$, so $E(r) = (\mathbf{r}=2\mathbf{e}_0)(r - a^2=r)$. For $a \to 0$, the field inside a uniformly charged solid rod in Problem 31(b) is recaptured.

Problem

33. A long, thin wire carries a uniform line charge density l = -6.8 mC/m. It is surrounded by a thick concentric cylindrical shell of inner radius 2.5 cm and outer radius 3.5 cm. What uniform volume charge density in the shell will result in zero electric field outside the shell?

Solution

In order to have $\mathbf{E} = 0$ outside the shell, it is only necessary for the charge per unit length of shell to cancel that of the wire, i.e., $\mathbf{I}_{\text{shell}} = +6.8 \text{ mC/m}$ (see Gauss's law and Equation 24-8, with $q_{\text{enclosed}} = 0$). A uniform charge density which guarantees this is $\mathbf{r} = \mathbf{I}_{\text{shell}} \ell = V$, where V is the volume of length ℓ of shell. Thus, $\mathbf{r} = \mathbf{I}_{\text{shell}} \ell = \mathbf{p}(r_2^2 - r_1^2)\ell = (6.8 \text{ mC/m}) = \mathbf{p}(3.5^2 - 2.5^2) \times 10^{-4} \text{ m}^2 = 3.61 \times 10^{-3} \text{ C/m}^3$.

Problem

34. A square nonconducting plate measures 4.5 m on a side and carries charge spread uniformly over its surface. The electric field 10 cm from the plate and not near an edge has magnitude 430 N/C and points toward the plate. Find (a) the surface charge density on the plate and (b) the total charge on the plate. (c) What is the electric field strength 20 cm from the plate.

Solution

We assume that the field due to the surface charge on the plate has plane symmetry (at least for the points considered in this problem), so that $E = s = 2e_0$ (positive away from the surface and negative toward it). Then (a) $s = 2e_0E = 2(8.85 \times 10^{-5})$

 10^{-12} N · m²/C²)(-430 N/C) = -7.61 nC/m², (b) $q = sA = (-7.61 \text{ nC/m}^2)(4.5 \text{ m})^2 = -154 \text{ nC}$, and (c) E = -430 N/C (*E* is independent of distance, as long as the distance is small enough to justify approximate plane symmetry).

Problem

35. If you "painted" positive charge on the floor, what surface charge density would be necessary in order to suspend a 15-*m*C, 5.0-g particle above the floor?

Solution

A positive surface charge density s, on the floor, would produce an approximately uniform electric field upward of $E = s=2e_0$, at points near the floor and not near an edge. The field needed to balance the weight of a particle, of mass m and charge q, is given by mg = qE, therefore $s = 2e_0E = 2e_0mg=q = 2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(5 \times 10^{-3} \text{ kg}) \times (9.8 \text{ m/s}^2)=(15 \times 10^{-6} \text{ C}) = 57.8 \text{ nC/m}^2$.

Problem

36. A slab of charge extends infinitely in two dimensions and has thickness d in the third dimension, as shown in Fig. 24-46. The slab carries a uniform volume charge density \mathbf{r} . Find expressions for the electric field strength (a) inside and (b) outside the slab, as functions of the distance x from the center plane.

Solution

If the slab were really infinite, the electric field would be everywhere normal to it (the *x* direction) and symmetrical about the center plane. (b) Gauss's law, applied to the surface superposed on Fig. 24-46, gives, for points outside the slab $(|x| > \frac{1}{2}d)$, $EA + EA = rdA=e_0$, or $E = rd=2e_0$ (equivalent to a sheet with s = rd). (a) For points inside the slab $(|x| \le \frac{1}{2}d)$, $2EA = r2xA=e_0$, or $E = rx=e_0$. E is directed away from (toward) the central plane for positive (negative) charge density.



FIGURE 24-46 Problem 36 Solution.

Problem

37. Figure 24-47 shows sections of three infinite flat sheets of charge, each carrying surface charge density with the same magnitude *s*. Find the magnitude and direction of the electric field in each of the four regions shown.



FIGURE 24-47 Problem 37.

Solution

The field from each sheet has magnitude $s=2e_0$ and points away from the positive sheets and toward the negative sheet. Take the *x* axis perpendicular to the sheets, to the right in Fig. 24-47. Superposition gives the field in each of the four regions, as shown.

First sheet:	=	$\overleftarrow{-s \hat{i}=2e_0}$	(+)	$s\hat{i}=2e_0$	(+)	sî= 2 →	-)	\underline{s} î =2 e_0
Second sheet		$-s\hat{i}=2e_0$	(+) (+)	$-s\hat{\mathbf{i}}=2e_0$	(+) (+)	$s\hat{i}=2e_0$	(-) (-)	s î= 2 e ₀
beeond sheet		sî-?e	(+)	sî-?a.	(+)	sî-?e.	(-)	-sî-?e.
Third sheet:		$\xrightarrow{312\mathbf{c}_0}$	(+) (+)	$\xrightarrow{\mathbf{31-2}\mathbf{c}_0}$	(+) (+)	$\xrightarrow{\mathbf{31-2}\mathbf{c}_0}$	(-) (-)	\leftarrow
Sum:		$-\boldsymbol{s}\hat{\boldsymbol{i}}=2\boldsymbol{e}_0$	(+)	$s\hat{i}=2e_0$	(+)	$\underbrace{3\mathbf{s}\mathbf{\hat{i}}=2\mathbf{e}_{0}}_{\mathbf{i}}$	(-)	$s\hat{i}=2e_0$
			(+)		(+)		(-)	

Section 24-5: Fields of Arbitrary Charge Distributions

Problem

38. A rod 50 cm long and 1.0 cm in radius carries a 2.0-*m*C charge distributed uniformly over its length. What is the approximate magnitude of the electric field (a) 4.0 mm from the rod surface, not near either end, and (b) 23 m from the rod?

Solution

(a) Close to the rod, but far from either end, the rod appears infinite, so Example 24-4 gives $E \bigvee I=2pe_0r$, or:

$$E = 2(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2 \text{ mC}=0.5 \text{ m})=(1.4 \text{ cm}) = 5.14 \times 10^6 \text{ N/C}$$

(b) Far away $(r \grave{A} \ell)$, the rod appears like a point charge, so

$$E \frac{1}{4} kq = r^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2 \text{ mC}) = (23 \text{ m})^2 = 34.0 \text{ N/C}.$$

Problem

39. A nonconducting square plate 75 cm on a side carries a uniform surface charge density. The electric field strength 1 cm from the plate, not near an edge, is 45 kN/C. What is the approximate field strength 15 m from the plate?

Solution

The electric field strength close to the plate (1 cm i 75 cm) has approximate plane symmetry ($E = s = 2e_0$), so the charge on the plate is $q = sA = 2e_0 EA = 2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(45 \text{ kN/C})(0.75 \text{ m})^2 = 448 \text{ nC}$. Very far from the plate

(15 m Å 0.75 m), the field strength is like that from a point charge, $E = kq = r^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(448 \text{ nC}) \times (15 \text{ m})^{-2} = 17.9 \text{ N/C}.$

Problem

40. Two circular plates 10 cm in diameter and 2.0 mm apart carry equal but opposite charges ±0.50 mC distributed uniformly over their facing surfaces. What is the electric field strength (a) between the plates but not near either edge? (b) 2.5 m from the plates on a plane passing midway between them? *Hint* for (b): See Example 23-6.

Solution

(a) Since the plate separation is much smaller than their size (2 mm i 10 cm), the electric field between them (but not near an edge) has approximate plane symmetry. The field of each plate has magnitude $s=2e_0 = q=2e_0A$ and is directed from the positive to the negative plate, therefore the total field there is $E = s=2e_0 + s=2e_0 = s=e_0 = q=e_0A$, in that direction, where q and A are the magnitude of the charge and area of one plate. Numerically, $E = (0.50 \text{ mC}) \div (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)p(5 \text{ cm})^2 = 7.19 \text{ MN/C}$. (b) Very far from both plates (2.5 m Å 0.1 m), the field is approximately like that from a point dipole. In a direction perpendicular to the dipole moment vector, Equation 23-7a gives a field strength of $E = kp=y^3 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.50 \text{ mC})(2 \text{ mm})=(2.5 \text{ m})^3 = 0.576 \text{ N/C}.$

Problem

41. The electric field strength on the axis of a uniformly charged disk is given by $E = 2\mathbf{p}k\mathbf{s}(1 - x = \sqrt{x^2 + a^2})$, with \mathbf{s} the surface charge density, a the disk radius, and x the distance from the disk center. If a = 20 cm, (a) for what range of x values does treating the disk as an infinite sheet give an approximation to the field that is good to within 10%? (b) For what range of x values is the point-charge approximation good to 10%?

Solution

(Note: The expression given, for the field strength on the axis of a uniformly charged disk, holds only for positive values of *x*.) (a) For small *x*, using the field strength of an infinite sheet, $E_{sheet} = s = 2e_0 = 2pks$, produces a fractional error less than 10% if $|E_{sheet} - E| = E < 0.1$. Since $E_{sheet} > E$, this implies that $E_{sheet} = E < 1.1$ or $2pks = 2pks(1 - x = \sqrt{x^2 + a^2}) < 1.1$. The steps in the solution of this inequality are: $1.1x < 0.1\sqrt{x^2 + a^2}$, $1.21x^2 < 0.01(x^2 + a^2)$, $x < a\sqrt{0.01 = 1.20} = 9.13 \times 10^{-2} a$. For a = 20 cm, x < 1.83 cm. (b) For large *x*, the point charge field, $E_{pt} = kq = x^2 = kpsa^2 = x^2$, is good to 10% for $|E_{pt} - E| = E < 0.1$. The solution of this inequality is simplified by defining an angle *f*, such that $\cos f = x = \sqrt{x^2 + a^2}$ and $\tan f = a = x$. In terms of *f*, one finds $E = 2pks(1 - \cos f)$, $E_{pt} = kps \tan^2 f$, and $E_{pt} = E = \tan^2 f = 2(1 - \cos f)$. Furthermore, $\tan^2 f = \sin^2 f = \cos^2 f = (1 - \cos f)(1 + \cos f) = \cos^2 f$, so $E_{pt} = E = (1 + \cos f) = 2\cos^2 f$. The range $0 \le x < \infty$ corresponds to $0 < f \le p = 2$, so $E_{pt} = E > 1$ and the inequality becomes $E_{pt} = E = (1 + \cos f) \neq 2\cos^2 f < 1.1$, or $2.2\cos^2 f - \cos f - 1 > 0$. The quadratic formula for the positive root gives $\cos f > (1 + \sqrt{1 + 8.8}) = 4.4 = 0.939$, or $f < 20.2^\circ$. This implies $x = a = \tan f > a = \tan 20.2^\circ = 2.72 a$. For a = 20 cm, x > 54.5 cm.

Problem

42. A nonconducting square 2.0 cm on a side carries a 45-nC charge spread uniformly over its surface. The *x* axis runs through the plate center, perpendicular to the plate, with x = 0 at the plate center. A -45-nC point charge is at x = 5.0 cm. Find approximate values for the electric field strength on the *x* axis at (a) x = 1.0 mm; (b) x = 4.8 cm; (c) x = 2.5 m. *Hint* for (c): Consult Example 23-6.

Solution

(a) Very close to the square (whose center is at x = 0), the field of the square is dominant, and this is

approximately that

of an infinite sheet. Then $E \ k E_{sq} = s = 2e_0 = 2pkq = (2 \text{ cm})^2 = 2p(9 \times 45 \times 10^4 \text{ N} \cdot \text{cm}^2/\text{C}) = (2 \text{ cm})^2 = 6.36 \text{ MN/C}.$ (The field of the point charge, $kq = (4.9 \text{ cm})^2 = 0.17 \text{ MN/C}$, is less and can be neglected in estimating the total field strength.) (b) Close to the point charge, its field dominates and that of the square can be ignored. Then

 $E \frac{1}{2} E_{pt} = kq = (0.2 \text{ cm})^2 = 101 \text{ MN/C.}$ (c) Far from both charges, the field is approximately that of their dipole moment. On the axis of the dipole, Equation 23-5b gives a field strength of $E \frac{1}{2} 2kp = x^3 = 2kq(5 \text{ cm}) = (2.5 \text{ m})^3 = 2.59 \text{ N/C.}$

Section 24-6: Gauss's Law and Conductors

Problem

43. What is the electric field strength just outside the surface of a conducting sphere carrying surface charge density 1.4 mC/m^2 ?

Solution

At the surface of a conductor, $E = \mathbf{s} = \mathbf{e}_0$ (positive away from the surface), or $(1.4 \text{ mC/m}^2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)^{-1} = 158 \text{ kN/C}$ in this problem.

Problem

44. Calculate the acceleration of a proton at the surface of a conductor carrying surface charge density 0.60 C/m^2 .

Solution

The acceleration of a proton $(eE=m_p)$ at the surface of a conductor (where $E = s=e_0$) is $a = es=e_0m_p = (1.6 \times 10^{-19} \text{ C}) \times (0.60 \text{ C/m}^2)=(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.67 \times 10^{-27} \text{ kg}) = 6.50 \times 10^{18} \text{ m/s}^2$.

Problem

45. A net charge of 5.0 **m**C is applied on one side of a solid metal sphere 2.0 cm in diameter. After electrostatic equilibrium is reached, what are (a) the volume charge density inside the sphere and (b) the surface charge density on the sphere? Assume there are no other charges or conductors nearby. (c) Which of your answers depends on this assumption, and why?

Solution

(a) The electric field within a conducting medium, in electrostatic equilibrium, is zero. Therefore, Gauss's law implies that the net charge contained in any closed surface, lying within the metal, is zero. (b) If the volume charge density is zero within the metal, all of the net charge must reside on the surface of the sphere. If the sphere is electrically isolated, the charge will be uniformly distributed (i.e., spherically symmetric), so $s = Q = 4pR^2 = (5 \text{ mC}) = 4p(1 \text{ cm})^2 = 3.98 \times 10^{-3} \text{ C/m}^2$. (c) Spherical symmetry for s depends on the proximity of other charges and conductors.

Problem

46. A point charge +q lies at the center of a spherical conducting shell carrying a net charge $\frac{3}{2}q$. Sketch the field lines both inside and outside the shell, using 8 field lines to represent a charge of magnitude q.

Solution

The field inside the shell is just due to the point charge (8 lines of force radiating outward). The field outside is like that of

a point charge $q + \frac{3}{2}q = \frac{5}{2}q$ (20 lines of force radiating outward). (Note: there is a charge -q, spread uniformly over the inner surface of the shell, and the field inside the conducting material is zero.)



Problem 46 Solution.

Problem

47. A 250-nC point charge is placed at the center of an uncharged spherical conducting shell 20 cm in radius. (a) What is the surface charge density on the outer surface of the shell? (b) What is the electric field strength at the shell's outer surface?

Solution

(a) There is a non-zero field outside the shell, because the net charge within is not zero. Therefore, there is a surface charge density $\mathbf{s} = \mathbf{e}_0 E$ on the outer surface of the shell, which is uniform, if we ignore the possible presence of other charges and conducting surfaces outside the shell. Gauss's law (with reasoning similar to Example 24-7) requires that the charge on the shell's outer surface is equal to the point charge within, so $\mathbf{s} = q=4\mathbf{p}R^2 = 250 \text{ nC}=4\mathbf{p}(0.20 \text{ m})^2 = 497 \text{ nC/m}^2$. (b) Then the field strength at the outer surface is $E = \mathbf{s}=\mathbf{e}_0 = 56.2 \text{ kN/C}$.

Problem

48. A point charge is placed at the center of an uncharged spherical conducting shell of inner radius 2.5 cm and outer radius 4.0 cm (Fig. 24-48). As a result, the outer surface of the shell acquires a surface charge density s = 71 nC/cm². Find (a) the value of the point charge and (b) the surface charge density on the inner wall of the shell.

Solution

Since the shell was uncharged, the total charge on the system (which is spread uniformly over the shell's outer surface) is equal to the point charge, q, and the charge on the inner surface of the shell is opposite to the charge on the outer surface. Thus, (a) $q = q_{outer} = 4\mathbf{p}(4 \text{ cm})^2(71 \text{ nC/cm}) = 14.3 \text{ mC}$, and (b) $\mathbf{s}_{inner} = -\mathbf{s}_{outer}(R_{outer}=R_{inner})^2 = -(71 \text{ nC/cm}^2) \times (4=2.5)^2 = -182 \text{ nC/cm}^2$.

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FIGURE 24-48 Problem 48 Solution.

Problem

49. An irregular conductor containing an irregular, empty cavity carries a net charge *Q*. (a) Show that the electric field inside the cavity must be zero. (b) If you put a point charge inside the cavity, what value must it have in order to make the surface charge density on the outer surface of the conductor everywhere zero?

Solution

(a) When there is no charge inside the cavity, the flux through any closed surface within the cavity (S_1) is zero, hence so is the field. (b) If the surface charge density on the outer surface (and also the electric field there) is to vanish, then the net charge inside a gaussian surface containing the conductor (S_2) is zero. Thus, the point charge in the cavity must equal -Q. (Note: The argument in part (a) depends on the conservative nature of the electrostatic field (see Section 25-1), for then positive flux on one part of S_1 canceling negative flux on another part is ruled out.)



Problem 49 Solution.

Problem

50. A neutral dime is placed in a uniform electric field of 6.2×10^5 N/C, with its faces perpendicular to the field. (a) What is the approximate charge density on the faces of the dime? (b) What is the total charge on each face? (Measure a dime!)

Solution

(a) If we assume that the uniform external field is unaffected by the presence of the dime, then Equation 24-11 gives $\mathbf{s} = \pm \mathbf{e}_0 E = \pm (6.2 \times 10^5 \text{ N/C}) \pm (36\mathbf{p} \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) = \pm 5.48 \text{ mC/m}^2$ (the field points away from the positively charged face). (b) Direct measurement shows that the diameter of a dime is about 18 mm, therefore $q = \mathbf{sp}R^2 = \pm (5.48 \text{ mC/m}^2)\mathbf{p}(9 \text{ mm})^2 = \pm 1.40 \text{ nC}$.

Problem

51. A total charge of 18 **m**C is applied to a thin, square metal plate 75 cm on a side. Find the electric field strength near the plate's surface.

Solution

The net charge of 18 mC must distribute itself over the outer surface of the plate, in accordance with Gauss's law for conductors. The outer surface consists of two plane square surfaces on each face, plus the edges and corners. Symmetry arguments imply that for an isolated plate, the charge density on the faces is the same, but not necessarily uniform because the edges and corners also have charge. If the plate is thin, we could assume that the edges and corners have negligible charge and that the density on the faces is approximately uniform. Then the surface charge density is the total charge divided by the area of both faces, s = 18 mC=2(75 cm)² = 16.0 mC, and the field strength near the plate (but not near an edge) is $E = s = e_0 = 1.81$ MN/C.

Problem

52. Two closely spaced parallel metal plates carry surface charge densities $\pm 95 \text{ nC/m}^2$ on their facing surfaces, with no charge on their outer surfaces. Find the electric field strength (a) between the plates and (b) outside the plates. Treat the plates as infinite in extent.

Solution

The last paragraph of Section 24-6 explains why (a) between the plates, $E = s = e_0 = (95 \text{ nC/m}^2) = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 10.7 \text{ kN/C}$, and (b) outside, but not near an edge, E = 0.

Problem

53. A conducting sphere 2.0 cm in radius is concentric with a spherical conducting shell with inner radius 8.0 cm and outer radius 10 cm. The small sphere carries 50 nC charge and the shell has no net charge. Find the electric field strength (a) 1.0 cm, (b) 5.0 cm, (c) 9.0 cm, and (d) 15 cm from the center.

Solution

If we assume the two-conductor system is isolated and in electrostatic equilibrium, then the field has spherical symmetry. Gauss's law requires that the field inside the conducting material be zero (for $0 \le r < 2$ cm and 8 cm < r < 10 cm in this problem), and that, since the shell is neutral, the field elsewhere is like that from a point charge of 50 mC located at the center of symmetry (r = 0). Thus, (a) E(1 cm) = 0, (b) $E(5 \text{ cm}) = kq = r^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(50 \text{ mC}) = (5 \text{ cm})^2 = 180 \text{ kN/C}$, (c) E(9 cm) = 0, and (d) $E(15 \text{ cm}) = kq = r^2 = (\frac{1}{9})E(5 \text{ cm}) = 20 \text{ kN/C}$.

Problem

54. A coaxial cable consists of an inner wire and a concentric cylindrical outer conductor (Fig. 24-49). If the conductors carry equal but opposite charges, show that there is no surface charge density on the *outside* of the outer conductor.

Solution

Assume line symmetry, and apply Gauss's law, as in Equation 24-8, to the outer cylindrical conducting surface, $2\mathbf{p} r\ell E_{surf} = q_{enclosed} = \mathbf{e}_0$. Since the conductors in the cable carry opposite charges of equal magnitude, there is zero charge enclosed, so the field and the charge density there ($\mathbf{s} = \mathbf{e}_0 E_{surf}$) vanish.

Paired Problems

Problem

55. A point charge -q is at the center of a spherical shell carrying charge +2q. That shell, in turn, is concentric with a larger shell carrying charge $-\frac{3}{2}q$. Draw a cross section of this structure, and sketch the electric field lines using the convention that 8 lines correspond to a charge of magnitude q.

Solution

The field from the given charges is spherically symmetric, so (from Gauss's law) is like that of a point charge, located at the center, with magnitude equal to the net charge enclosed by a sphere of radius equal to the distance to the field point. Thus, $E = -kq = r^2$ inside the first shell (8 lines radially inward), $E = +kq = r^2$ between the first and second shells (8 lines radially outward), and $E = -kq = 2r^2$ outside the second shell (4 lines radially inward).



Problem 55 Solution.

Problem

56. A point charge -q is at the center of a spherical shell carrying charge $-\frac{3}{2}q$. That shell, in turn, is concentric with a larger shell carrying charge +2q. Draw a cross section of this structure, and sketch the electric field lines using the convention that 8 lines correspond to a charge of magnitude q.

Solution

The field is spherically symmetric, as in the previous problem, but now $E = -kq = r^2$ inside the first shell (8 lines inward), $E = -5kq = 2r^2$ between the shells (20 lines inward), and $E = -kq = 2r^2$ outside (4 lines inward).



Problem 56 Solution.

Problem

57. A point charge q is at the center of a spherical shell of radius R carrying charge 2q spread uniformly over its surface. Write expressions for the electric field strength at (a) $\frac{1}{2}R$ and (b) 2R.

Solution

As explained in Example 24-3, (a) for $r = \frac{1}{2}R < R$, $q_{\text{enclosed}} = q$ and $E = kq = (\frac{1}{2}R)^2 = 4kq = R^2$, and (b) for r = 2R > R, $q_{\text{enclosed}} = q + 2q$ and $E = 3kq = (2R)^2 = 3kq = 4R^2$.

Problem

58. A point charge q is at the center of a spherical shell of radius R carrying charge 5q. At what other distance does the electric field have the same value it does at a point halfway from the center to the shell?

Solution

At $r = \frac{1}{2}R$, $E = 4kq=R^2$, as in part (a) of the previous problem. Outside the shell, $E(r) = 6kq=r^2$, which equals this for $r^2 = 6R^2=4$, or $r = \sqrt{1.5} R = 1.22R$.

Problem

59. A long, thin hollow pipe 4.0 cm in diameter carries charge at a density of -2.6 mC/m, uniformly distributed over the pipe. It is concentric with 10-cm diameter pipe carrying +2.6 mC/m, also uniformly distributed. Find the magnitude of the electric field at (a) 0.50 cm, (b) 3.5 cm, and (c) 12 cm from the axis of the pipes.

Solution

Assume the electric field has line symmetry, and apply Gauss's law to a coaxial cylindrical surface of radius r. The result is Equation 24-8 set equal to $\mathbf{1}_{enclosed} \notin \mathbf{e}_0$, so $E(r) = \mathbf{1}_{enclosed} = 2\mathbf{p}\mathbf{e}_0 r$. (a) At r = 0.5 cm < 2 cm (inside inner pipe), $\mathbf{1}_{enclosed}$ is zero and so is E. (b) At 2 cm < r = 3.5 cm < 5 cm (between the pipes), $\mathbf{1}_{enclosed}$ is just the inner pipe, so $E = (-2.6 \text{ mC/m}) = 2\mathbf{p}\mathbf{e}_0(3.5 \text{ cm}) = -1.34 \text{ MN/C}$. (The minus sign means the direction of E is radially inward toward the axis of the pipes; the magnitude is the absolute value of E.) (c) At r = 12 cm > 5 cm (outside outer pipe), $\mathbf{1}_{enclosed}$ and E are again zero, since the pipes have opposite linear charge densities.

Problem

60. Two concentric hollow pipes are 5.0 cm and 12 cm in diameter, respectively. Both carry uniformly distributed electric charges. The electric field 4.0 cm from their common axis is 630 kN/C, radially outward. The field 10 cm from their common axis is 126 kN/C, radially outward. (a) Find the linear charge densities on the two pipes. (b) How would the electric field strengths at 4.0 cm and 10 cm change if the charge density on the outer pipe were doubled?

Solution

(a) As in the previous problem, between the pipes, $E = 630 \text{ kN/C} = \mathbf{l}_{\text{in}} = 2\mathbf{p}\mathbf{e}_0(4 \text{ cm})$, and outside the pipes, $E = 126 \text{ kN/C} = (\mathbf{l}_{\text{in}} + \mathbf{l}_{\text{out}}) = 2\mathbf{p}\mathbf{e}_0(10 \text{ cm})$. Thus, $\mathbf{l}_{\text{in}} = 2\mathbf{p}\mathbf{e}_0(4 \text{ cm})(630 \text{ kN/C}) = 1.40 \text{ mC/m}$, and $\mathbf{l}_{\text{out}} = 2\mathbf{p}\mathbf{e}_0(10 \text{ cm})(126 \text{ kN/C}) - \mathbf{l}_{\text{in}} = -0.700 \text{ mC/m}$. (b) The field at 4 cm only depends on \mathbf{l}_{in} , so there is no change in E there. Since $\mathbf{l}_{\text{out}} = -\frac{1}{2}\mathbf{l}_{\text{in}}$ (or $\mathbf{l}_{\text{in}} + 2\mathbf{l}_{\text{out}} = 0$), there would be zero field outside the pipes if the charge density on the outer pipe were doubled.

Problem

61. An early (and incorrect) model for the atom pictured its positive charge as spread uniformly throughout the spherical atomic volume. For a hydrogen atom of radius 0.0529 nm, what would be the electric field due to such a distribution of

positive charge (a) 0.020 nm from the center and (b) 0.20 nm from the center?

Solution

(a) Inside a uniformly charged spherical volume, $E = kQr = R^3 = (9 \text{ GN} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})(0.02 \text{ nm}) = (0.0529 \text{ nm})^3 = 195 \text{ GN/C}$ (see Equation 24-7). (b) Outside, the field is like that of a point charge, $E = kQ = r^2 = (9 \text{ GN} \cdot \text{m}^2/\text{C}^2)$ $(1.6 \times 10^{-19} \text{ C}) = (0.2 \text{ nm})^2 = 36.0 \text{ GN/C}$ (see Equation 24-6).

Problem

62. A solid sphere of radius R carries a charge spread uniformly throughout its volume. At what point outside the sphere is the electric field strength equal to that at a point halfway from the center to the edge? Express your answer as a distance from the center.

Solution

From Equations 24-6 and 7, $kQ(\frac{1}{2}R)=R^3 = kQ=r^2$ implies $r = \sqrt{2}R$.

Problem

63. A sphere of radius 2a has a hole of radius a, as shown in Fig. 24-50. The solid portion carries a uniform volume charge density r. Find an expression for the electric field strength within the solid portion, as a function of the distance r from the center.

Solution

From Gauss's law and Equation 24-5, $E = q_{\text{enclosed}} = 4pe_0r^2$, where q_{enclosed} is the charge within a spherical gaussian surface of radius *r* about the center of symmetry. For a < r < 2a, $q_{\text{enclosed}} = \mathbf{r}V = \frac{4}{3}\mathbf{pr}(r^3 - a^3)$, so $E = (\mathbf{r} = 3e_0)(r - a^3 = r^2)$.



FIGURE 24-50 Problem 63 Solution.

Problem

64. Repeat the previous problem, now considering that the figure represents the cross section of a thick cylindrical pipe.

Solution

For cylindrical symmetry, $E = \mathbf{l}_{enclosed} = 2\mathbf{p}\mathbf{e}_0 r$, and if a < r < 2a, $\mathbf{l}_{enclosed} = \mathbf{r}\mathbf{p}(r^2 - a^2)$. Thus, $E = (\mathbf{r} = 2\mathbf{e}_0)(r - a^2 = r)$.

Supplementary Problems

Problem

65. Repeat Problem 10 for the case $\mathbf{E} = E_0 \sum_{i=1}^{2} \hat{\mathbf{k}}_i$.

Solution

Since the electric field depends only on y, break up the square in Fig. 24-42 (see Problem 10) into strips of area $d\mathbf{A} = \pm a \, dy \, \hat{\mathbf{k}}$, of length a parallel to the x axis and width dy, the normal to which could be $\pm \hat{\mathbf{k}}$. The electric flux through the square is

$$\Phi = \sum_{\text{square}} \mathbf{E} \cdot d\mathbf{A} = \pm \sum_{a} E_0 \sum_{a}^{b} \sum_{a}^{b} a \, dy = \pm \sum_{a} \sum_{b} \sum_{a} y^2 \, dy = \pm \frac{1}{3} E_0 a^2.$$

Problem

66. The volume charge density inside a solid sphere of radius *a* is given by $\mathbf{r} = \mathbf{r}_0 \mathbf{r} = a$, where \mathbf{r}_0 is a constant. Find (a) the total charge and (b) the electric field strength within the sphere, as a function of distance *r* from the center.

Solution

(a) The charge inside a sphere of radius $r \le a$ is $q(r) = \mathbf{Z} \mathbf{r} dV$. For volume elements, take concentric shells of radius r and thickness dr, so $dV = 4\mathbf{p}r^2 dr$. Then

$$q(r) = 4\mathbf{p} \sum r^2 dr = 4\mathbf{p}(\mathbf{r}_0=a) \sum r^3 dr = \mathbf{p}\mathbf{r}_0 r^4=a.$$

For r = a, the total charge is pr_0a^3 . (b) For spherical symmetry, Gauss's law and Equation 24-5 give $4pr^2E(r) = q(r)=e_0 = pr_0r^4=e_0a$, or $E(r) = r_0r^2=4e_0a$.

Problem

67. A proton is released from rest 1.0 cm from a large sheet carrying a surface charge density of -24 nC/m^2 . How much later does it strike the sheet?

Solution

The proton is accelerated toward the sheet by an electric field in that direction. For the field, we can use that from an infinite plane sheet (assuming we are not near an edge) so $E = s=2e_0$ and the acceleration $a = eE=m_p$ is uniform. Starting from rest, the proton travels to the sheet in time

$$t = \sqrt{2(x - x_0)} = a = \sqrt{4e_0 m_p (x - x_0)} = s = \sqrt{\frac{4(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.67 \times 10^{-27} \text{ kg})(1 \text{ cm})}{(1.6 \times 10^{-19} \text{ C})(24 \text{ nC/m}^2)}} = 392 \text{ ns}$$

Problem

68. Figure 24-51 shows a rectangular box with sides 2a and length ℓ surrounding a line of charge with uniform line charge density I. The line passes directly through the center of the box faces. Using an expression for the field of a line charge, integrate over strips of width dx as shown to find the electric flux through one face of the box. Multiply by 4 to get the total flux through the box, and show that your result is consistent with Gauss's law.

Solution

The electric field on the top surface of the box has magnitude $l=2pe_0r$, and direction radially away from the line of charge. The flux through a strip of width dx at position x is $d\Phi = E \cos q \ell dx = (l=2pe_0r)(a=r)\ell dx$, where $r = \sqrt{x^2 + a^2}$, so

$$\Phi_{\rm top} = \sum_{a} \left[\frac{a\ell}{2pe_0} \frac{dx}{ka^2 + x^2} \right] = \left[\frac{a\ell}{2pe_0} \frac{dx}{ka^2 + x^2} \right]$$

From symmetry, the flux through the whole box is four times this, or $l \ell = e_0$, which equals $q_{\text{enclosed}} = e_0$.



FIGURE 24-51 Problem 68 Solution.

Problem

69. Repeat Problem 36 for the case when the charge density in the slab is given by $\mathbf{r} = \mathbf{r}_0 |\mathbf{x} = d|$, where \mathbf{r}_0 is a constant.

Solution

Gauss's law, plane symmetry, and Equation 24-9 can be used to find the electric field strength, but we must integrate to get the charge enclosed by the gaussian surface. We use charge elements that are thin parallel sheets of the same area as the face of the gaussian surface and of thickness dx, as shown. (a) Inside the slab (|x| < d=2), $2EA = e_0^{-1} \mathbf{Z}_x \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x (-x) \, dx + \mathbf{Z} x \, dx = \mathbf{r}_0 A x^2 = e_0^{-1} \mathbf{A}$, hence $E = \mathbf{r}_0 x^2 = 2\mathbf{e}_0 d$. (b) Outside, $2EA = \mathbf{e}_0^{-1} \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, dx = (\mathbf{r}_0 A = e_0^{-1}) \mathbf{Z}_x^{-2} \mathbf{r} A \, d$

 $(\mathbf{r}_0 A = \mathbf{e}_0 d)(\mathbf{Z}_{d=2}(-x) dx + \mathbf{Z}_{\mathbf{r}}^{=2} x dx) = (\mathbf{r}_0 A = \mathbf{e}_0 d)(d=2)^2$, hence $E = \mathbf{r}_0 d = 8\mathbf{e}_0$. (This is equivalent, of course, to the field strength outside an infinite sheet with $\mathbf{s} = \mathbf{r}_0 d = 4$.)



FIGURE 24-45 Problem 69 Solution.

Problem

70. The charge density within a uniformly charged sphere of radius *R* is given by $\mathbf{r} = \mathbf{r}_0 - ar^2$, where \mathbf{r}_0 and *a* are constants, and *r* is the distance from the center. Find an expression for *a* such that the electric field outside the sphere is zero.

Solution

The field outside the sphere will be zero if the total charge within the volume is zero. Thus, using thin concentric shells as charge elements, we require that $0 = \mathbf{Z}_{\text{sphere}} \mathbf{r} \, dV = \mathbf{Z}(\mathbf{r}_0 - ar^2) 4\mathbf{p} r^2 \, dr = 4\mathbf{p}[\frac{1}{3}\mathbf{r}_0 R^3 - \frac{1}{5}aR^5]$, or $a = 5\mathbf{r}_0 = 3R^2$.

Problem

71. A small object of mass *m* and charge *q* is attached by a thread of length ℓ to a large, flat, nonconducting plate carrying a uniform surface charge density *s* with the same sign as *q* (Fig. 24-52). If the object is displaced slightly sideways from its equilibrium, show that it undergoes simple harmonic motion with period $T = 2p\sqrt{2e_0m\ell - qs}$. Assume the gravitational force is negligible.



FIGURE 24-52 Problem 71.

Solution

The Coulomb force on the particle is approximately uniform and greater than its weight. Therefore, the net upward force field on the particle is $F_{\text{net}}=m = (q=n)E - g$, and since it is tethered by the string, it oscillates like an upside-down pendulum with period $T = 2p\sqrt{\ell=(qE=m-g)}$. If we neglect gravity and use $E = s=2e_0$ for an infinite sheet as an approximation, then $T = 2p\sqrt{2e_0m\ell=qs}$.

Problem

72. An infinitely long nonconducting rod of radius *R* carries a volume charge density given by $\mathbf{r} = \mathbf{r}_0(\mathbf{r}=\mathbf{R})$, where \mathbf{r}_0 is a constant. Find the electric field strength (a) inside and (b) outside the rod, as functions of the distance *r* from the rod axis.

Solution

Line symmetry, Equation 24-8, and Gauss's law give a field strength of $E = \mathbf{l}_{enclosed} = 2\mathbf{p}\mathbf{e}_0 r$, where $\mathbf{l}_{enclosed} = \mathbf{z}_0 \mathbf{r} dV$ is the charge within a unit length of coaxial cylindrical surface of radius *r*, and $dV = 2\mathbf{p}r dr$ is the volume element for a unit length of thin shell with this surface. (a) For r < R (inside rod), $\mathbf{l}_{enclosed} = \mathbf{z}_0(2\mathbf{p}\mathbf{r}_0 = R)r^2 dr = 2\mathbf{p}\mathbf{r}_0 r^3 = 3R$, hence $E = \mathbf{r}_0 r^2 = 3\mathbf{e}_0 R$. (b) For r > R (outside rod), $\mathbf{l}_{enclosed} = \mathbf{z}_0(2\mathbf{p}\mathbf{r}_0 = R)r^2 dr = 2\mathbf{p}\mathbf{r}_0 R^2 = 3\mathbf{e}_0 r$.

Problem

73. A thick spherical shell of inner radius *a* and outer radius *b* carries a charge density given by $\mathbf{r} = \frac{ce^{-r=a}}{r^2}$, where *a* and *c* are constants. Find expressions for the electric field strength for (a) r < a, (b) a < r < b, and (c) r > b.

Solution

Spherical symmetry, Equation 24-5 and Gauss's law give a field strength of $E(r) = q_{\text{enclosed}} = 4pe_0 r^2$, where

 $q_{\text{enclosed}} = \mathbf{Z} \mathbf{r} dV$ is the charge within a concentric spherical surface of radius *r*, and $dV = 4\mathbf{p}r^2 dr$ is the volume element for a thin shell with this surface. (a) For r < a, $q_{\text{enclosed}} = 0$ hence E(r) = 0. (b) For $a \le r \le b$, $q_{\text{enclosed}} = 4\mathbf{p}c \mathbf{Z}_a e^{-r = a} dr = 4\mathbf{p}ac(e^{-1} - e^{-r = a})$ hence $E(r) = ac(e^{-1} - e^{-r = a}) = \mathbf{e}_0 r^2$. (c) For r > b, $q_{\text{enclosed}} = 4\mathbf{p}c \mathbf{Z}_a e^{-r = a} dr$ hence $E(r) = ac(e^{-1} - e^{-r = a}) = \mathbf{e}_0 r^2$.

Problem

74. A solid sphere of radius *R* carries a nonuniform volume charge density given by $\mathbf{r} = \mathbf{p}^2 \mathbf{r}_0 \sin(\mathbf{p} \mathbf{r} = \mathbf{R})$, where *r* is the distance from the center and \mathbf{r}_0 is a positive constant. Find the magnitude and direction of the electric field at the sphere's surface.

Solution

The electric field strength due to a spherically symmetric distribution of charge, with surface at radius *R*, is $E(R) = (4pe_0R^2)^{-1} \sum_{r=1}^{\infty} r(4pr^2 dr) = (e_0R^2)^{-1} \sum_{r=1}^{\infty} rr^2 dr$. With $r = p^2 r_0 \sin(pr=R)$ given,

$$E(R) = \left[\frac{\mathbf{r}_{0}^{2} \mathbf{r}_{0}}{\mathbf{q}_{0}^{2} \mathbf{r}^{2}} \sin(\mathbf{p} \mathbf{r} = \mathbf{R}) d\mathbf{r} = \left[\frac{\mathbf{r}_{0}^{2} \mathbf{R}}{\mathbf{p} \mathbf{e}_{0}} \mathbf{k}^{2} u \sin u - (u^{2} - 2) \cos u \right]_{0}^{p} = \left[\frac{\mathbf{r}_{0}^{2} \mathbf{R}}{\mathbf{p} \mathbf{e}_{0}} \mathbf{k}^{2} - 4 \right]$$

(Let $u = \mathbf{p} \mathbf{r} = \mathbf{R}$ and see any standard table of integrals.) Of course, $\mathbf{E} = E\hat{\mathbf{r}}$, so positive $E(\mathbf{R})$ is directed radially outward.

Problem

75. A solid sphere of radius *R* carries a uniform volume charge density *r*. A hole of radius *R*=2 occupies a region from the center to the edge of the sphere, as shown in Fig. 24-53. Show that the electric field everywhere in the hole points horizontally and has magnitude $rR=6e_0$. *Hint:* Treat the hole as a superposition of two charged spheres of opposite charge.

Solution

A large solid sphere can be considered to be the superposition of the sphere with a cavity plus a small solid sphere filling the cavity, both with uniform charge density \mathbf{r} . The electric field inside the solid spheres is $\mathbf{rr}=3\mathbf{e}_0$, where \mathbf{r} is a vector from the center of each sphere to the field point P, in both (see Equation 24-7). For the large sphere, whose center we take at the origin, $\mathbf{r} = \mathbf{r}_P$, and for the small sphere, whose center is at $\frac{1}{2}R\mathbf{\hat{i}}$, $\mathbf{r} = \mathbf{r}_P - \frac{1}{2}R\mathbf{\hat{i}}$. Therefore, \mathbf{E} (large sphere) = \mathbf{E} (sphere with cavity) + \mathbf{E} (small sphere), or $\mathbf{rr}_P=3\mathbf{e}_0 = \mathbf{E} + \mathbf{r}(\mathbf{r}_P - \frac{1}{2}R\mathbf{\hat{i}})=3\mathbf{e}_0$. Thus, $\mathbf{E} = \mathbf{rR}\mathbf{\hat{i}}=6\mathbf{e}_0$, that is, for any point inside the cavity, the electric field of the sphere with the cavity is uniform (with direction parallel to the line between the centers of the sphere and cavity). (Note that this result holds for any size spherical cavity if one replaces $\frac{1}{2}R\mathbf{\hat{i}}$ with the vector to the center of the cavity.)



FIGURE 24-53 Problem 75 Solution.

Problem

76. You're 5.0 m from a charge distribution and you measure an electric field strength of 850 N/C. At 2.5 m the field strength has increased to about 3.4 kN/C. When you're 5.0 mm from the center of the distribution the field strength is 42.5 MN/C, and it increases to 85 MN/C at 2.5 mm. Describe the distribution as fully as you can, including its shape, any dimensions you can find, its total charge, and any appropriate charge density.

Solution

Since 850 N/C = $\frac{1}{4} \times 3.40$ kN/C and 5 m = 2 × 2.5 m, one might guess that the field strength varies like $1 = r^2$ and that these distances are large compared to the size of the charge distribution. Then these relatively smaller fields are approximately like point charge fields, and $q = (850 \text{ N/C})(5 \text{ m})^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2 = \text{C}^2) = 2.36 \text{ mC}$ is the total charge. At close range, presumably very near the distribution, (42.5 MN/C)(5 mm) = (85.0 MN/C)(2.5 mm), so the field strength might vary like 1=r. The field near a long straight wire has this dependence if $l = (42.5 \text{ MN/C} \times 50 \text{ mm})=2(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) = 11.8 \text{ mC/m}$. Thus, the distribution could be a charged straight wire with total charge 2.36 mC and length $\ell = q = l = (2.36 = 11.8) \text{ m} = 20 \text{ cm}$. (Of course, two points don't uniquely determine the field dependence.)

Problem

77. Two flat, parallel, closely spaced metal plates of area 0.080 m² carry total charges of -2.1 mC and +3.8 mC. Find the surface charge densities on the inner and outer faces of each plate.

Solution

If the thickness and separation of the plates is small compared to their lateral dimensions ($\sqrt{0.08 \text{ m}^2}$ ½ 28 cm), we can assume that the electric field near the plates (edge effects neglected) is uniform and normal to the plates. (Of course, far away, the field goes like $\exists r^2$.) The general case, where the plates carry arbitrary charges q_1 and q_2 , can be viewed as the superposition of three simpler cases: a neutral plate, a charged plate, and two oppositely charged plates (see last paragraph in Section 24-6). In each of the three regions, left of, between, and right of the plates (the thickness, assumed negligible, does not add to any region), the electric field is the sum of three contributions, as shown. The final diagram, together with Equation 24-11 (which gives $\mathbf{s} = \mathbf{e}_0 \mathbf{E}$ at each surface), shows that the charge densities on the outer surfaces are equal, $\mathbf{s}_{out} = \frac{1}{2}(q_1 + q_2)=A$, and that the charge densities on the inner surfaces are equal and opposite, $\mathbf{s}_{in} = \pm \frac{1}{2}(q_2 - q_1)=A$. Numerically, $\mathbf{s}_{out} = \frac{1}{2}(-2.1 + 3.8) \mathbf{m}C=0.08 \text{ m}^2 = 10.6 \mathbf{m}C/\text{m}^2$, and $\mathbf{s}_{in} = \pm \frac{1}{2}[3.8 - (-2.1)] \mathbf{m}C=0.08 \text{ m}^2 = \pm 36.9 \mathbf{m}C/\text{m}^2$. (Note: the direction of the fields is shown for positive total charge, with the more positive plate on the right.)



Problem 77 Solution.

Problem

78. Since the gravitational force of a point mass goes as l=r², the gravitational field g also obeys a form of Gauss's law.
(a) Formulate this law, and (b) use it to find an expression for the gravitational field strength *within* the Earth, as a function of distance r from the center. Treat Earth as a sphere of uniform density.

Solution

(a) Newton's law for the gravitational field, $\mathbf{g} = \mathbf{F} = m = -GM\hat{\mathbf{r}} = r^2$, and Coulomb's law for the electrostatic field, $\mathbf{E} = q\hat{\mathbf{r}} = 4p\boldsymbol{e}_0 r^2$ (both for point sources) are identical when $q = 4p\boldsymbol{e}_0$ is replaced by -GM. By analogy to Equation 24-4, the expression of Gauss's law for gravity is $\mathbf{g} \cdot d\mathbf{A} = -4p GM_{\text{enclosed}}$. (b) Extending the analogy to Equation 24-7, we find $g(r) = -GM_E r = R_E^3$ for the gravitational field at a point $r < R_E$ within a uniform Earth. (The minus sign means g is radially inward, as appropriate to an attractive force.)